BRIEF COMMUNICATIONS

TEMPERATURE DISTRIBUTION IN A CENTRIFUGED LIQUID

R. S. Kuznetskii

Inzhenerno-Fizicheskii Zhurnal, Vol. 11, No. 2, pp. 258-260, 1966

UDC 536.25

Let there be a layer of liquid of known mass covering the inside solid surface of a cylinder and at rest relative to it (resulting, for example, from the action of a centrifugal force after protracted rotation of the cylinder about its axis).

When the temperatures of the solid surface and of the medium in direct contact with the free surface of the liquid are constant, we have for the liquid the heat conduction equation with boundary conditions for the temperature

$$\frac{d}{dr}\left(\lambda r \frac{dt}{dr}\right) = 0, \quad \lambda r \frac{dt}{dr}\bigg|_{r=a,b} = \alpha_1 a \Delta t_1 = \alpha_2 b \Delta t_2 \quad (1)$$

and an integral equation for the mass

$$2\pi \int_b^a \frac{rdr}{v} = m. \quad (2)$$

For constant λ we obtain a logarithmic temperature distribution

$$t = t_1 + \Theta \ln \rho, \quad 0 < \xi \leqslant \rho \leqslant 1, \quad \Delta t = -\Theta \ln \xi; \quad (3)$$

its parameters t_1 and Θ, as well as the thickness of the liquid layer (or ξ) require determination.

Taking the α_i to be constants, we have, from the boundary conditions

$$t_1 = t_{e1} - \frac{\Delta t_e \xi}{\xi(1 - \mathrm{Nu} \ln \xi) + \beta}, \quad t_2 = t_{e2} + \frac{\beta \Delta t_e}{\xi(1 - \mathrm{Nu} \ln \xi) + \beta},$$

$$\Theta = \mathrm{Nu}\, \Delta t_1 = \frac{\mathrm{Nu}\, \Delta t_e \xi}{\xi(1 - \mathrm{Nu} \ln \xi) + \beta}. \quad (4)$$

In what follows we shall understand t_1, t_2 and Θ to have these expressions.

We restrict the examination to an imcompressible liquid with constant thermal expansion coefficient

$$v = v_0 \exp(\delta t), \quad v_0 = \mathrm{const}. \quad (5)$$

Then (2) leads to a transcendental equation for ξ:

$$(1 - \xi^{2 - \mathrm{Nu}\vartheta}) \exp(\vartheta - \vartheta_1) = n(2 - \mathrm{Nu}\,\vartheta),$$

$$\vartheta = \varepsilon \frac{\xi}{\xi(1 - \mathrm{Nu} \ln \xi) + \beta}. \quad (6)$$

We note that there are trivial special cases:

$$\text{when } \alpha_1 = 0 \quad t = t_{e1}, \text{ when } \alpha_2 = 0 \quad t = t_{e2},$$

$$\text{when } \Delta t_e = 0 \quad t = t_e, \quad (7)$$

where $\xi = \xi_0$ for these. Assuming these to be exceptions, we have

$$0 < \frac{1}{\Delta t_e}\{\Delta t, \Delta t_1, \Delta t_2\} < 1. \quad (8)$$

When $\delta = 0$

$$\xi = \sqrt{1 - 2n}. \quad (9)$$

When δ is sufficiently small (then $\xi_0^2 > 0$)

$$|\varepsilon| \ll \min\left[1; \frac{1}{\mathrm{Nu}};\right.$$

$$\left. 2\xi_0 \frac{\beta + \xi_0(1 - \mathrm{Nu} \ln \xi_0)}{n(\mathrm{Nu} + 2)\exp(\vartheta_1) + \mathrm{Nu}\, \xi_0^2 \ln \xi_0} \min(1; |\ln \xi_0|)\right], \quad (10)$$

and ξ differs from ξ_0 by the small quantity

$$\eta = \frac{\varepsilon}{2} \frac{n(\mathrm{Nu} + 2)\exp(\vartheta_1) + \mathrm{Nu}\, \xi_0^2 \ln \xi_0}{\beta + \xi_0(1 - \mathrm{Nu} \ln \xi_0)}, \quad (11)$$

whereupon

$$\Theta = \frac{\mathrm{Nu}\, \Delta t_e}{\beta + \xi_0(1 - \mathrm{Nu} \ln \xi_0)}\left[\xi_0 + \frac{\beta + \mathrm{Nu}\, \xi_0}{\beta + \xi_0(1 - \mathrm{Nu} \ln \xi_0)} \eta\right],$$

$$t_1 = t_{e1} - \frac{\Delta t_e}{\beta + \xi_0(1 - \mathrm{Nu} \ln \xi_0)}\left[\xi_0 + \frac{\beta + \mathrm{Nu}\, \xi_0}{\beta + \xi_0(1 - \mathrm{Nu} \ln \xi_0)} \eta\right]. \quad (12)$$

Postulating that ξ is sufficiently small

$$\xi \ll \min\left(1; \beta; \frac{\beta}{|\varepsilon|}\right), \quad \xi |\ln \xi| \ll \frac{\beta}{\mathrm{Nu} \cdot |\varepsilon|} \quad (13)$$

(the liquid occupies almost the whole inside of the cylinder), we obtain

$$\xi = -\frac{\beta}{\varepsilon} \frac{\xi_0^2}{\mathrm{Nu}\, n \exp(\vartheta_1) + 1}, \quad (14)$$

and then

$$\Theta = -\frac{\mathrm{Nu}}{\delta} \frac{\xi_0^2}{\mathrm{Nu}\, n \exp(\vartheta_1) + 1}, \quad t_1 = t_{e1} + \frac{1}{\delta} \frac{\xi_0^2}{\mathrm{Nu}\, n \exp(\vartheta_1) + 1}. \quad (15)$$

We note in passing that there is an inequality

$$\exp\left[\frac{\varepsilon\xi}{\xi(1 - \mathrm{Nu} \ln \xi) + \beta}\right] <$$

$$< \frac{2n}{1 - \xi^2} \exp(\vartheta_1) < \exp\left[\frac{\xi(1 - \mathrm{Nu} \ln \xi)}{\xi(1 - \mathrm{Nu} \ln \xi) + \beta}\right] \text{ for } \Delta t_e > 0 \quad (16)$$

stemming from the evident condition $v_{min} < v_0(1 - \xi^2)/2n < v_{max}$, and an inequality of the opposite sense when $\Delta t_e < 0$. From these it may be seen that the parameter ξ_0^2 may be both positive and negative, being zero when $2n = \exp(\vartheta_1)$. It may be seen, for example, from (14), that the opposite signs of ξ_0^2 and Δt_e correspond to the case being examined of (13) with small ξ.

For sufficiently small n

$$n \ll \exp\left(\frac{\varepsilon}{\beta + 1} - \vartheta_1\right) \cdot \min\left[1; \frac{\beta + 1}{\mathrm{Nu} - 1}; \frac{(\beta + 1)^2}{(\beta + \mathrm{Nu})|\varepsilon|}\right] \quad (17)$$

(the layer of liquid is thin) we obtain for μ the quadratic equation

$$\mu\left[1 - \frac{\beta + \mathrm{Nu}}{(\beta + 1)^2} \varepsilon\mu\right] = n \exp\left(\vartheta_1 - \frac{\varepsilon}{\beta + 1}\right), \quad (18)$$

whence we have, approximately

$$\mu = n \exp\left(\vartheta_1 - \frac{\varepsilon}{\beta + 1}\right), \quad (19)$$

and then

$$\Theta = \frac{\mathrm{Nu}}{\beta + 1} \Delta t_e \left(1 - \frac{\beta + \mathrm{Nu}}{\beta + 1} \mu\right),$$

$$t_1 = t_{e1} - \frac{\Delta t_e}{\beta + 1}\left(1 - \frac{\beta + \mathrm{Nu}}{\beta + 1} \mu\right). \quad (20)$$

The total enthalpy of the cylindrical layer of liquid per unit length of generator is

$$I = 2\pi a^2 c \int_{\xi}^{1} \frac{t}{v} \rho \, d\rho = \frac{2\pi a^2 c}{v_0 \delta (2 - \mathrm{Nu}\,\vartheta)} \frac{\exp(\vartheta - \vartheta_1)}{\xi(1 - \mathrm{Nu} \ln \xi) + \beta} \times$$

$$\times \Big\{ \big[[\vartheta_1 (1 - \ln \xi) - \varepsilon] \xi + \beta \vartheta_1 \big] (1 - \xi^{2 - \mathrm{Nu}\,\vartheta}) +$$

$$+ \mathrm{Nu}\, \varepsilon \xi \frac{\xi^{2 - \mathrm{Nu}\,\vartheta} [1 - (2 - \mathrm{Nu}\,\vartheta) \ln \xi] - 1}{2 - \mathrm{Nu}\,\vartheta} \Big\}; \tag{21}$$

when $\delta = 0$

$$I = \frac{\pi a^2 c}{v_0} \{ 2nt_1 - \Theta [2n + (1 - 2n) \ln (1 - 2n)] \}, \tag{22}$$

where t_1 and Θ must be taken with regard for (9).

NOTATION

c is the specific heat, $J \cdot kg^{-1} \cdot degree^{-1}$; I and m are the total enthalpy, $J \cdot m^{-1}$, and mass, $kg \cdot m^{-1}$ of the cylindrical layer of liquid with unit length of generator; ν is the specific volume, $m^3 \cdot kg^{-1}$; r is the radius vector, equal to a on the solid cylindrical surface, and b on the free surface of the liquid, m; t is the temperature, °K; $\Theta = dt/d\ln r$; α, δ and λ are the coefficients of heat transfer, $W \cdot m^{-2} \cdot degree^{-1}$, thermal expansion, $degree^{-1}$, and thermal conductivity, $W \cdot m^{-1} \cdot degree^{-1}$; $\Delta t_e = t_{e1} - t_{e2}$, $\Delta t = t_1 - t_2$, $\Delta t_1 = t_{e1} - t_1$, $\Delta t_2 = t_2 - t_{e2}$ are the temperature drops; $n = m\nu_0/2\pi a^2 < 1/2$, $\mathrm{Nu} = \alpha_1 a/\lambda$ is the Nusselt number, $\varepsilon = \delta \Delta t_e$, $\vartheta - \delta \Delta t_1$, $\vartheta_1 = \delta t_{e1}$, $\rho = r/a$, $\xi = 1 - \mu = b/a$, $\xi_0^2 = 1 - 2n \exp(\vartheta_1)$, $h = \xi - \xi_0$ are dimensionless quantities. Subscripts 1 and 2 relate, respectively, to r = a and r = b; e relates to the external media.

9 March 1966

"Giprostal" Institute, Khar'kov

USE OF INVERSE METHODS OF HEAT CONDUCTION TO DETERMINE CONDITIONS OF UNSTEADY HEAT TRANSFER

A. G. Temkin, A. G. Gromyko, and R. V. Amitonov

Inzhenerno-Fizicheskii Zhurnal, Vol. 11, No. 2, pp. 261-263, 1966

UDC 536.212

1. Determination of radiative-convective heat transfer on a heated cylinder [1, 2]. This was carried out in investigations of bimetallization of sliding bearings used in shipbuilding [3]. A bushing of St 15 steel had diameters of 82 and 62 mm, and length 100 mm. A batch of OTsS 6-6-3 bronze was fused in an electric arc and uniformly distributed over the inside surface of the bushing by centrifugal force. The rotating product was cooled on the outside with air, and on the inside with nitrogen. The heat balance of this kind of an unsteady technical process may be set up only with the aid of inverse methods of heat conduction [1, 4, 5]. Pt versus Pt-Rh thermocouples were embedded at distances of 31 and 38.5 mm from the axis, and their emf's were recorded on a EKVO-III-8711 strip potentiometer. The temperatures, measured to an accuracy of 5° at a speed of 750 rpm, are shown in Table 1, which also gives the temperature of the cooled surface, calculated from formula (3.18) of reference [1]. The first three terms of the series were used to calculate this.

The thermal diffusivity a [6] was taken at 1273° K, equal to 0.065 cm^2/sec, and the thermal conductivity was $\lambda = 0.2375$ W/cm · °K. By differentiation with respect to the coordinate $\nu = r/r_1$ we calculated the heat fluxes

$$q = -2\pi\nu\lambda \, \partial t(\nu, \tau)/\partial \nu, \tag{1}$$

the heating $\nu = \nu_i = 1$, dissipation $\nu = \nu_e = 1.323$, and accumulation $q_a = q_i - q_e$.

The graph of the modified Biot number

$$\mathrm{Bi} = [2\pi\nu_e \, \partial t(\nu_e, \tau)/\partial \nu]/[t(\nu_e, \tau) - t_e] \tag{2}$$

outwardly resembles the curve for q_i (Fig. 1), increasing from 1.2 to 5, and has a stationary value of 2.22.

2. Experimental determination of the unsteady contact heat transfer of a heated hollow sphere. St 45 steel spheres of diameter 61.00 and 159.75 mm were used. The sphere was positioned in sand saturated with water in a large container. The inside surface of the sphere was insulated with a 4 mm layer of asbestos from a spherical nichrome coil to which voltage was applied. Chromel-alumel thermocouples were located at $r_1 = 36.9$, $r_2 = 49.5$ and $r_3 = 75.5$ mm from the center. They were insulated with mineral fiber and located in ϕ 1.5 mm steel tubes. The temperature was recorded on a 28KVT potentiometer with scale from 273 to 473° K for the thermocouple used. The measured temperatures are shown in Table 2.

From these temperatures, in accordance with formula (5.2) of reference [1], we calculated the thermal diffusivity, which proved to be equal to 0.1292 cm^2/sec at a mean temperature of 405° K. The handbook values at 373 and 423° K are 0.1308 and 0.1269 cm^2/sec, and 0.1288 cm^2/sec was taken. From the temperatures at the edge points the temperature field of the sphere was reconstructed, according to the formula (4.17) of reference [1]:

$$q = -4\pi\nu^2\lambda \, r_1 \partial t(\nu, \tau)/\partial \nu. \tag{3}$$

The calculated power supplied to the internal surface of the sphere is $\nu_i = 0.8266$, and that dissipated through the outside surface is $\nu_e = 2.1653$. For St 45 steel [6, 7], $\lambda = 0.474$ W/cm · °K. These quantities are shown in Fig. 2, in addition to the accumulated power q_a.

Table 1

Temperatures of Internal Points and Cylinder Surface

τ, min	T, °K			τ, min	T, °K		
	at point 1	at point 2	on cylinder surface		at point 1	at point 2	on cylinder surface
1.0	698	633	625.5	4.5	1458	1343	1311.9
1.5	963	868	851.2	5.0	1463	1373	1348.7
2.0	1023	1023	1003.3	5.5	1478	1388	1363.3
2.5	1283	1123	1084.2	6.0	1493	1403	1377.9
3.0	1353	1198	1158.6	6.5	1493	1408	1383.9
3.5	1408	1258	1218.6	7.0	1498	1413	1388.5
4.0	1443	1313	1278.5				